PapaCambridge

Cambridge International AS & A Level

MATHEMATICS (9709) P3

TOPIC WISE QUESTIONS + ANSWERS | COMPLETE SYLLABUS

PapaCambridge

Chapter 5

Integration

154. 9709_s20_qp_31 Q: 5

(a) Find the quotient and remainder when $2x^3 - x^2 + 6x + 3$ is divided by $x^2 + 3$. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(b) Using your answer to part **(a)**, find the exact value of $\int_1^3 \frac{2x^3 - x^2 + 6x + 3}{x^2 + 3}\,dx$. [5]

155. 9709_s20_qp_31 Q: 7

Let $f(x) = \dfrac{\cos x}{1 + \sin x}$.

(a) Show that $f'(x) < 0$ for all x in the interval $-\frac{1}{2}\pi < x < \frac{3}{2}\pi$. [4]

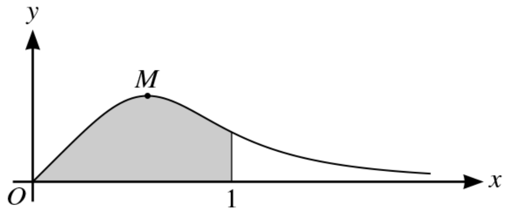
(b) Find $\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} f(x)\,dx$. Give your answer in a simplified exact form. [4]

156. 9709_s20_qp_32 Q: 3

Find the exact value of

$$\int_1^4 x^{\frac{3}{2}} \ln x \, dx.$$ [5]

157. 9709_s20_qp_32 Q: 6



The diagram shows the curve $y = \dfrac{x}{1 + 3x^4}$, for $x \geqslant 0$, and its maximum point M.

(a) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [4]

(b) Using the substitution $u = \sqrt{3}x^2$, find by integration the exact area of the shaded region bounded by the curve, the x-axis and the line $x = 1$. [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

158. 9709_s20_qp_33 Q: 2

Find the exact value of $\int_0^1 (2 - x)e^{-2x}\,dx$. [5]

159. 9709_s20_qp_33 Q: 7

Let $f(x) = \dfrac{2}{(2x-1)(2x+1)}$.

(a) Express $f(x)$ in partial fractions. [2]

....

....

....

....

....

....

....

....

....

(b) Using your answer to part **(a)**, show that

$$\left(f(x)\right)^2 = \frac{1}{(2x-1)^2} - \frac{1}{2x-1} + \frac{1}{2x+1} + \frac{1}{(2x+1)^2}.$$ [2]

....

....

....

....

....

....

....

....

....

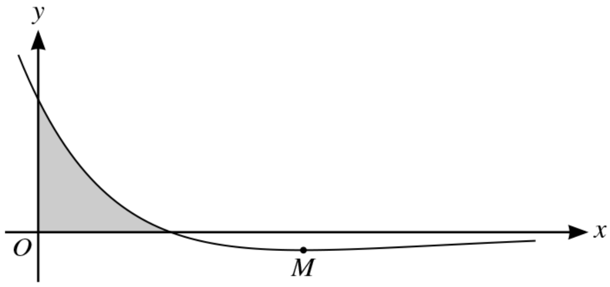
....

....

....

(c) Hence show that $\int_1^2 (f(x))^2\,dx = \frac{2}{5} + \frac{1}{2}\ln\left(\frac{5}{9}\right)$. [5]

160. 9709_w20_qp_31 Q: 10



The diagram shows the curve $y = (2 - x)e^{-\frac{1}{2}x}$, and its minimum point M.

(a) Find the exact coordinates of M. [5]

(b) Find the area of the shaded region bounded by the curve and the axes. Give your answer in terms of e. [5]

161. 9709_w20_qp_32 Q: 9

Let $f(x) = \dfrac{7x + 18}{(3x + 2)(x^2 + 4)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_0^2 f(x)\,dx$. [6]

162. 9709_m19_qp_32 Q: 4

Show that $\int_1^4 x^{-\frac{3}{2}} \ln x \, dx = 2 - \ln 4$. [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

163. 9709_s19_qp_31 Q: 1

Use the trapezium rule with 3 intervals to estimate the value of

$$\int_0^3 |2^x - 4| \, dx.$$ [3]

164. 9709_s19_qp_32 Q: 8

Let $f(x) = \dfrac{10x + 9}{(2x + 1)(2x + 3)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^1 f(x)\,dx = \frac{1}{2}\ln\frac{9}{5} + \frac{1}{5}$. [5]

165. 9709_s19_qp_33 Q: 2

Show that $\int_0^{\frac{1}{4}\pi} x^2 \cos 2x \, dx = \frac{1}{32}(\pi^2 - 8)$. [5]

166. 9709_s19_qp_33 Q: 3

Let $f(\theta) = \dfrac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta}$.

(i) Show that $f(\theta) = \tan \theta$. [3]

(ii) Hence show that $\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} f(\theta)\, d\theta = \frac{1}{2}\ln\frac{3}{2}$. [4]

167. 9709_w19_qp_31 Q: 6

(i) By differentiating $\frac{\cos x}{\sin x}$, show that if $y = \cot x$ then $\frac{dy}{dx} = -\operatorname{cosec}^2 x$. [2]

(ii) Show that $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} x \operatorname{cosec}^2 x \, dx = \frac{1}{4}(\pi + \ln 4)$. [6]

168. 9709_w19_qp_31 Q: 8

Let $f(x) = \dfrac{x^2 + x + 6}{x^2(x+2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing full working, show that the exact value of $\int_1^4 f(x)\,dx$ is $\frac{9}{4}$. [5]

169. 9709_w19_qp_31 Q: 9

(i) By first expanding $\cos(2x + x)$, show that $\cos 3x \equiv 4\cos^3 x - 3\cos x$. [4]

(ii) Hence solve the equation $\cos 3x + 3\cos x + 1 = 0$, for $0 \leqslant x \leqslant \pi$. [2]

....

....

....

....

....

....

(iii) Find the exact value of $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \cos^3 x \, dx$. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

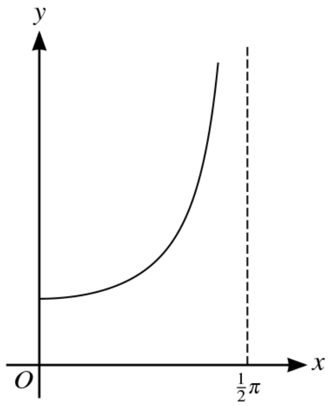
170. 9709_w19_qp_32 Q: 8

Let $f(x) = \dfrac{2x^2 + x + 8}{(2x - 1)(x^2 + 2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing full working, find $\int_1^5 f(x)\,dx$, giving the answer in the form $\ln c$, where c is an integer. [5]

171. 9709_w19_qp_33 Q: 8



The diagram shows the graph of $y = \sec x$ for $0 \leqslant x < \frac{1}{2}\pi$.

(i) Use the trapezium rule with 2 intervals to estimate the value of $\int_0^{1.2} \sec x \, dx$, giving your answer correct to 2 decimal places. [3]

....

....

....

....

....

....

....

(ii) Explain, with reference to the diagram, whether the trapezium rule gives an overestimate or an underestimate of the true value of the integral in part **(i)**. [1]

....

....

....

....

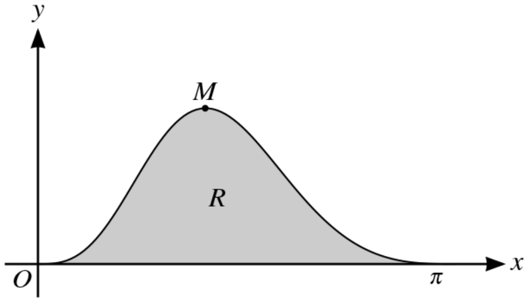
....

....

....

(iii) P is the point on the part of the curve $y = \sec x$ for $0 \leqslant x < \frac{1}{2}\pi$ at which the gradient is 2. By first differentiating $\dfrac{1}{\cos x}$, find the x-coordinate of P, giving your answer correct to 3 decimal places. [6]

172. 9709_w19_qp_33 Q: 10



The diagram shows the graph of $y = e^{\cos x} \sin^3 x$ for $0 \leqslant x \leqslant \pi$, and its maximum point M. The shaded region R is bounded by the curve and the x-axis.

(i) Find the x-coordinate of M. Show all necessary working and give your answer correct to 2 decimal places. [5]

(ii) By first using the substitution $u = \cos x$, find the exact value of the area of R. [7]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

173. 9709_m18_qp_32 Q: 1

Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{4}\pi} \sqrt{(1 - \tan x)}\, dx,$$

giving your answer correct to 3 decimal places. [3]

174. 9709_m18_qp_32 Q: 3

(i) Using the expansions of $\cos(3x + x)$ and $\cos(3x - x)$, show that

$$\tfrac{1}{2}(\cos 4x + \cos 2x) \equiv \cos 3x \cos x.$$ [3]

(ii) Hence show that $\int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} \cos 3x \cos x \, dx = \frac{3}{8}\sqrt{3}$. [3]

175. 9709_m18_qp_32 Q: 8

Let $f(x) = \dfrac{5x^2 + x + 27}{(2x+1)(x^2+9)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence find $\int_0^4 f(x)\,dx$, giving your answer in the form $\ln c$, where c is an integer. [5]

176. 9709_s18_qp_31 Q: 5

Let $I = \int_{\frac{1}{4}}^{\frac{3}{4}} \sqrt{\left(\frac{x}{1-x}\right)}\,dx$.

(i) Using the substitution $x = \cos^2\theta$, show that $I = \int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} 2\cos^2\theta\,d\theta$. [4]

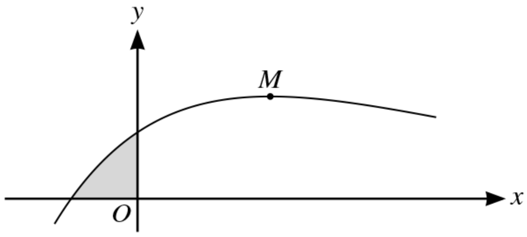
(ii) Hence find the exact value of I. [4]

177. 9709_s18_qp_32 Q: 4

(i) Show that $\dfrac{2\sin x - \sin 2x}{1 - \cos 2x} \equiv \dfrac{\sin x}{1 + \cos x}$. [4]

(ii) Hence, showing all necessary working, find $\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \frac{2\sin x - \sin 2x}{1 - \cos 2x}\, dx$, giving your answer in the form $\ln k$. [4]

178. 9709_s18_qp_32 Q: 8



The diagram shows the curve $y = (x + 1)e^{-\frac{1}{3}x}$ and its maximum point M.

(i) Find the x-coordinate of M. [4]

(ii) Find the area of the shaded region enclosed by the curve and the axes, giving your answer in terms of e. [5]

179. 9709_s18_qp_33 Q: 3

Showing all necessary working, find the value of $\int_0^{\frac{1}{6}\pi} x\cos 3x\,dx$, giving your answer in terms of π. [5]

180. 9709_s18_qp_33 Q: 7

Throughout this question the use of a calculator is not permitted.

(i) Express $\cos\theta + 2\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact values of R and $\tan\alpha$. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

(ii) Hence, showing all necessary working, show that $\int_0^{\frac{1}{4}\pi} \frac{15}{(\cos\theta + 2\sin\theta)^2}\,d\theta = 5$. [5]

....

....

....

....

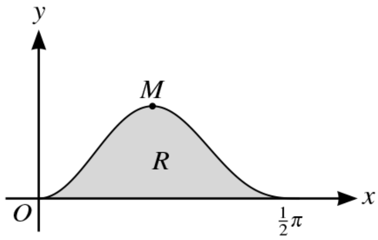
....

....

....

....

181. 9709_w18_qp_31 Q: 7



The diagram shows the curve $y = 5\sin^2 x\cos^3 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M. The shaded region R is bounded by the curve and the x-axis.

(i) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(ii) Using the substitution $u = \sin x$ and showing all necessary working, find the exact area of R. [4]

182. 9709_w18_qp_31 Q: 9

Let $\mathrm{f}(x) = \dfrac{6x^2 + 8x + 9}{(2 - x)(3 + 2x)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence, showing all necessary working, show that $\int_{-1}^{0} f(x)\,dx = 1 + \frac{1}{2}\ln\left(\frac{3}{4}\right)$. [5]

183. 9709_w18_qp_32 Q: 3

(i) Find $\int \frac{\ln x}{x^3}\,dx$. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(ii) Hence show that $\int_1^2 \frac{\ln x}{x^3}\,dx = \frac{1}{16}(3 - \ln 4)$. [2]

....

....

....

....

....

....

....

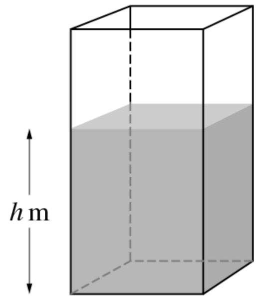
184. 9709_w18_qp_32 Q: 7

A curve has equation $y = \dfrac{3\cos x}{2 + \sin x}$, for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Find the exact coordinates of the stationary point of the curve. [6]

(ii) The constant a is such that $\int_0^a \frac{3\cos x}{2+\sin x}\,dx = 1$. Find the value of a, giving your answer correct to 3 significant figures. [4]

185. 9709_m17_qp_32 Q: 7



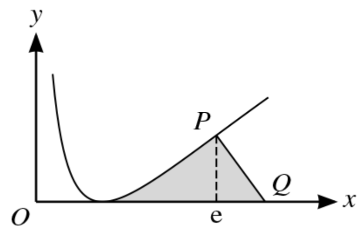
A water tank has vertical sides and a horizontal rectangular base, as shown in the diagram. The area of the base is $2\,\text{m}^2$. At time $t = 0$ the tank is empty and water begins to flow into it at a rate of $1\,\text{m}^3$ per hour. At the same time water begins to flow out from the base at a rate of $0.2\sqrt{h}\,\text{m}^3$ per hour, where h m is the depth of water in the tank at time t hours.

(i) Form a differential equation satisfied by h and t, and show that the time T hours taken for the depth of water to reach 4 m is given by

$$T = \int_0^4 \frac{10}{5 - \sqrt{h}}\, dh.$$ [3]

(ii) Using the substitution $u = 5 - \sqrt{h}$, find the value of T. [6]

186. 9709_m17_qp_32 Q: 10



The diagram shows the curve $y = (\ln x)^2$. The x-coordinate of the point P is equal to e, and the normal to the curve at P meets the x-axis at Q.

(i) Find the x-coordinate of Q. [4]

(ii) Show that $\int \ln x \, dx = x \ln x - x + c$, where c is a constant. [1]

(iii) Using integration by parts, or otherwise, find the exact value of the area of the shaded region between the curve, the x-axis and the normal PQ. [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

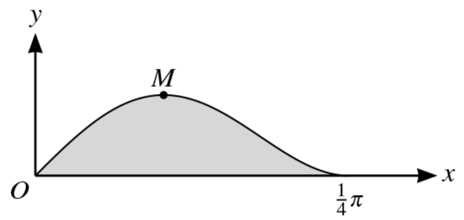
187. 9709_s17_qp_31 Q: 3

It is given that $x = \ln(1 - y) - \ln y$, where $0 < y < 1$.

(i) Show that $y = \dfrac{e^{-x}}{1 + e^{-x}}$. [2]

(ii) Hence show that $\int_0^1 y\,dx = \ln\left(\frac{2e}{e+1}\right)$. [4]

188. 9709_s17_qp_31 Q: 10



The diagram shows the curve $y = \sin x \cos^2 2x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$ and its maximum point M.

(i) Using the substitution $u = \cos x$, find by integration the exact area of the shaded region bounded by the curve and the x-axis. [6]

(ii) Find the x-coordinate of M. Give your answer correct to 2 decimal places. [6]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

189. 9709_s17_qp_32 Q: 7

(i) Prove that if $y = \dfrac{1}{\cos\theta}$ then $\dfrac{dy}{d\theta} = \sec\theta\tan\theta$. [2]

(ii) Prove the identity $\dfrac{1+\sin\theta}{1-\sin\theta} \equiv 2\sec^2\theta + 2\sec\theta\tan\theta - 1$. [3]

....

....

....

....

....

....

....

....

....

....

(iii) Hence find the exact value of $\int_0^{\frac{1}{4}\pi} \frac{1 + \sin\theta}{1 - \sin\theta}\,d\theta$. [4]

....

....

....

....

....

....

....

....

....

....

....

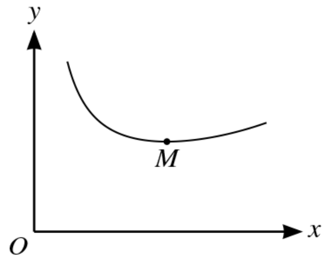
....

....

190. 9709_s17_qp_33 Q: 4

Find the exact value of $\int_0^{\frac{1}{2}\pi} \theta \sin \frac{1}{2}\theta \, d\theta$. [4]

191. 9709_s17_qp_33 Q: 7



The diagram shows a sketch of the curve $y = \dfrac{e^{\frac{1}{2}x}}{x}$ for $x > 0$, and its minimum point M.

(i) Find the x-coordinate of M. [4]

(ii) Use the trapezium rule with two intervals to estimate the value of

$$\int_1^3 \frac{e^{\frac{1}{2}x}}{x}\,dx,$$

giving your answer correct to 2 decimal places. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(iii) The estimate found in part **(ii)** is denoted by E. Explain, without further calculation, whether another estimate found using the trapezium rule with four intervals would be greater than E or less than E. [1]

....

....

....

....

....

....

192. 9709_s17_qp_33 Q: 9

Let $f(x) = \dfrac{3x^2 - 4}{x^2(3x + 2)}$.

(i) Express $f(x)$ in partial fractions. [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(ii) Hence show that $\int_1^2 f(x)\,dx = \ln\left(\frac{25}{8}\right) - 1$. [5]

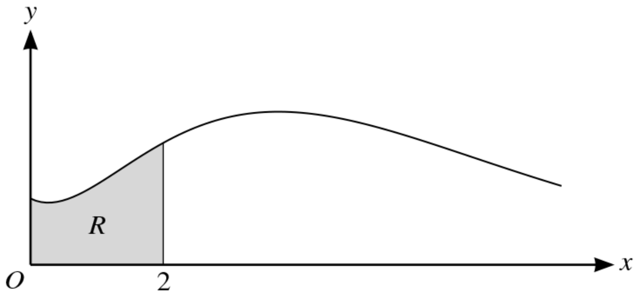
193. 9709_w17_qp_31 Q: 8

Let $f(x) = \dfrac{4x^2 + 9x - 8}{(x+2)(2x-1)}$.

(i) Express $f(x)$ in the form $A + \dfrac{B}{x+2} + \dfrac{C}{2x-1}$. [4]

(ii) Hence show that $\int_1^4 f(x)\,dx = 6 + \frac{1}{2}\ln(\frac{16}{7})$. [5]

194. 9709_w17_qp_31 Q: 9



The diagram shows the curve $y = (1 + x^2)e^{-\frac{1}{2}x}$ for $x \geqslant 0$. The shaded region R is enclosed by the curve, the x-axis and the lines $x = 0$ and $x = 2$.

(i) Find the exact values of the x-coordinates of the stationary points of the curve. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

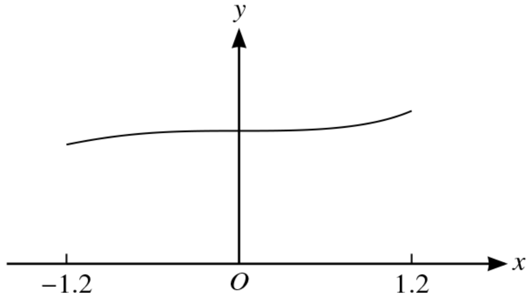
....

....

....

(ii) Show that the exact value of the area of R is $18 - \dfrac{42}{e}$. [5]

195. 9709_w17_qp_32 Q: 1



The diagram shows a sketch of the curve $y = \frac{3}{\sqrt{(9 - x^3)}}$ for values of x from −1.2 to 1.2.

(i) Use the trapezium rule, with two intervals, to estimate the value of

$$\int_{-1.2}^{1.2} \frac{3}{\sqrt{(9 - x^3)}}\, dx,$$

giving your answer correct to 2 decimal places. [3]

....

....

....

....

....

....

....

(ii) Explain, with reference to the diagram, why the trapezium rule may be expected to give a good approximation to the true value of the integral in this case. [1]

....

....

....

....

....

....

....

196. 9709_m16_qp_32 Q: 5

Let $I = \int_0^1 \frac{9}{(3+x^2)^2}\,dx$.

(i) Using the substitution $x = (\sqrt{3})\tan\theta$, show that $I = \sqrt{3}\int_0^{\frac{1}{6}\pi} \cos^2\theta\,d\theta$. [3]

(ii) Hence find the exact value of I. [4]

197. 9709_m16_qp_32 Q: 9

Let $\mathrm{f}(x) = \dfrac{3x^3 + 6x - 8}{x(x^2 + 2)}$.

(i) Express $\mathrm{f}(x)$ in the form $A + \dfrac{B}{x} + \dfrac{Cx + D}{x^2 + 2}$. [5]

(ii) Show that $\displaystyle\int_1^2 \mathrm{f}(x)\,\mathrm{d}x = 3 - \ln 4$. [5]

198. 9709_s16_qp_31 Q: 2

Find the exact value of $\int_0^{\frac{1}{2}} x\mathrm{e}^{-2x}\,\mathrm{d}x$. [5]

199. 9709_s16_qp_32 Q: 3

Find the exact value of $\int_0^{\frac{1}{2}\pi} x^2 \sin 2x \, dx$. [5]

200. 9709_s16_qp_32 Q: 7

Let $f(x) = \dfrac{4x^2 + 7x + 4}{(2x + 1)(x + 2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\int_0^4 f(x)\,dx = 8 - \ln 3$. [5]

201. 9709_s16_qp_33 Q: 7

Let $I = \int_0^1 \frac{x^5}{(1+x^2)^3}\,dx$.

(i) Using the substitution $u = 1 + x^2$, show that $I = \int_1^2 \frac{(u-1)^2}{2u^3}\,du$. [3]

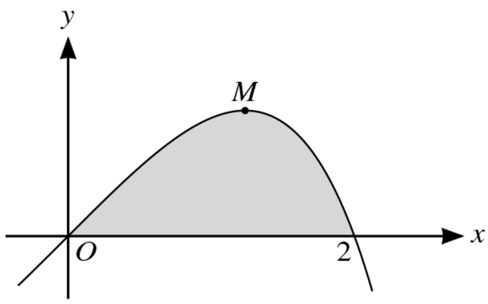
(ii) Hence find the exact value of I. [5]

202. 9709_w16_qp_31 Q: 5

(i) Prove the identity $\tan 2\theta - \tan\theta \equiv \tan\theta \sec 2\theta$. [4]

(ii) Hence show that $\int_0^{\frac{1}{6}\pi} \tan\theta \sec 2\theta \, d\theta = \frac{1}{2} \ln \frac{3}{2}$. [4]

203. 9709_w16_qp_31 Q: 7



The diagram shows part of the curve $y = (2x - x^2)e^{\frac{1}{2}x}$ and its maximum point M.

(i) Find the exact x-coordinate of M. [4]

(ii) Find the exact value of the area of the shaded region bounded by the curve and the positive x-axis. [5]

204. 9709_w16_qp_33 Q: 6

Let $I = \int_1^4 \frac{(\sqrt{x}) - 1}{2(x + \sqrt{x})}\, dx$.

(i) Using the substitution $u = \sqrt{x}$, show that $I = \int_1^2 \frac{u - 1}{u + 1}\, du$. [3]

(ii) Hence show that $I = 1 + \ln \frac{4}{9}$. [6]

205. 9709_s15_qp_31 Q: 2

Use the trapezium rule with three intervals to find an approximation to

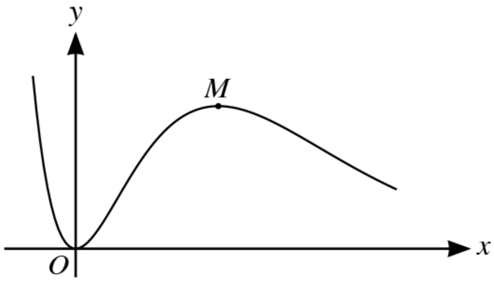
$$\int_0^3 |3^x - 10| \, dx.$$ [4]

206. 9709_s15_qp_31 Q: 5

(a) Find $\int (4 + \tan^2 2x)\, dx$. [3]

(b) Find the exact value of $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \frac{\sin(x + \frac{1}{6}\pi)}{\sin x}\, dx$. [5]

207. 9709_s15_qp_31 Q: 9



The diagram shows the curve $y = x^2 e^{2-x}$ and its maximum point M.

(i) Show that the x-coordinate of M is 2. [3]

(ii) Find the exact value of $\int_0^2 x^2 e^{2-x}\, dx$. [6]

208. 9709_s15_qp_32 Q: 1

Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{2}\pi} \ln(1 + \sin x)\, dx,$$

giving your answer correct to 2 decimal places. [3]

209. 9709_s15_qp_32 Q: 6

Let $I = \int_0^1 \frac{\sqrt{x}}{2 - \sqrt{x}}\, dx$.

(i) Using the substitution $u = 2 - \sqrt{x}$, show that $I = \int_1^2 \frac{2(2-u)^2}{u}\, du$. [4]

(ii) Hence show that $I = 8 \ln 2 - 5$. [4]

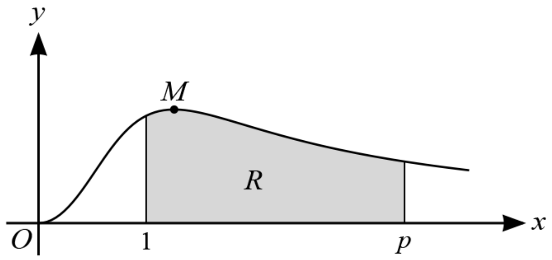
210. 9709_s15_qp_33 Q: 10

Let $f(x) = \dfrac{11x + 7}{(2x - 1)(x + 2)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\int_1^2 f(x)\,dx = \frac{1}{4} + \ln\left(\frac{9}{4}\right)$. [5]

211. 9709_w15_qp_31 Q: 10



The diagram shows the curve $y = \frac{x^2}{1+x^3}$ for $x \geqslant 0$, and its maximum point M. The shaded region R is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = p$.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) Calculate the value of p for which the area of R is equal to 1. Give your answer correct to 3 significant figures. [6]

212. 9709_w15_qp_33 Q: 5

Use the substitution $u = 4 - 3\cos x$ to find the exact value of $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{9\sin 2x}{\sqrt{(4 - 3\cos x)}}\,dx$. [8]

213. 9709_w15_qp_33 Q: 7

(i) Show that $(x+1)$ is a factor of $4x^3 - x^2 - 11x - 6$. [2]

(ii) Find $\int \frac{4x^2 + 9x - 1}{4x^3 - x^2 - 11x - 6} \, dx$. [8]